A SIMPLE COMPUTER MODEL FOR THE GROWTH OF LIGHT POLLUTION

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ABSTRACT

A simple model of the propagation and scattering of light in the atmosphere has been used to calculate the light pollution contributions from each city in southern Ontario and its vicinity. These were then summed to give the total artificial contribution to the background night-sky brightness in the region. A series of computer programs were used to generate digital and graphic maps of light pollution in southern Ontario from a list of the locations and populations of cities. By making simple assumptions about the population growth of cities and the increase, with time, in the amount of light generated by a city of a given size, the programs have been used to determine the future threat of light pollution to astronomical observations in the area.

Introduction. As light pollution has recently emerged as a serious threat to both amateur and professional astronomical observation, it is time to determine the present and future extent of the problem. Whether an observer has a fixed site or travels to better skies, an understanding of the extent and probable growth of the artificial sky background will allow an optimization of observing methods, programs and "strategies". Many observers, particularly amateurs, must travel some distance in order to be as far from city lights as possible when making serious observations. As such journeys are usually only for a night or weekend, it is important to know which sites have the darkest skies so that the ratio of travelling time to observing time may be minimized.

In the fall of 1974, the Observational Activities Committee of the Toronto Centre of the R.A.S.C. initiated the Sky Brightness Programme, with the goals of monitoring, studying and predicting the extent of light pollution in Ontario. Berry (1976) has presented the results of this project in terms of a map of sky brightness levels across southern Ontario. This map was computed on the basis of a simple model of light propagation and scattering from cities and towns. In this paper, the details of the computer calculations used to produce this map will be given along with calculations of future trends of light pollution.

The Static Model. Berry (1976) gives the following equation for the sky brightness resulting from artificial light generated by a city, based on work by Treanor (1973):

$$B = a\sqrt{p}\left(\frac{U}{D^2 + h^2} + \frac{V}{\sqrt{D^2 + h^2}}\right)\exp\left(-k\sqrt{D^2 + h^2}\right) \tag{1}$$

where B is the sky brightness, a is a proportionality constant, p is the city's population, h is the effective height of scattering, D is the distance from the observation point to the city, k is the absorption coefficient of the atmosphere, and U and V are empirical parameters. With B in units of tenth magnitude stars per square degree (called S_{10} units), and D in kilometers, the constants have the following values:

$a = 50.0$ S_{10} units per "root person"
$U = 2.59$ km^2
$V = .08$ km
$k = .026$ km^{-1}
$h = 2.4$ km

Given equation (1) and a list of the locations and populations of cities, one can calculate the expected sky brightness at a given point by simply summing the contributions of all the cities. Actually, only those that are sufficiently close and large to be significant contributors to the sky background at that point need be considered. The values of the constants are assumed to be the same from city to city. This assumption is reasonable over an area of fairly constant economic development and geography such as southern Ontario, but using the same constants for a different area such as California may produce erroneous results. In California, for example, the k value would likely be smaller due to the drier and more transparent atmosphere.

A computer program was written to evaluate the contributions to the sky background from each city in or near Ontario, on the basis of equation (1). An input file containing the locations and populations (1975) of about 300 cities was used to produce a quantized approximation to the distribution of artificial light over the province. Only cities and towns of 1000 or more population in Ontario, and American cities of ~5000 or greater population within 50–100 km of the Canadian border were used in the model. Ontario towns north of Sault Ste. Marie were ignored, as were most of the small towns in northeastern New York. The program was written in "C" to run under the UNIX operating system on a PDP-11/45 computer owned jointly by the Computer Research Facility and the Dynamic Graphics Project of the University of Toronto.

The output from this program was an array of numbers giving the brightness of the sky at each point on a grid covering the province. This array was then processed by a second program which created a "map" of the area on a graphics display. Figure 1a is a photograph of the display, showing a typ-

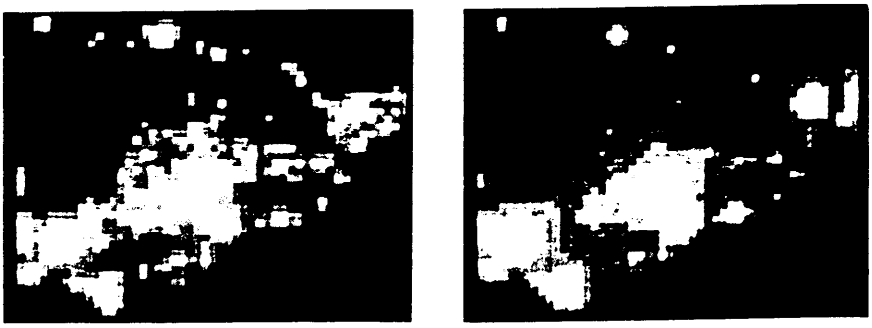

FIG. 1—Left (a): The static model for southern Ontario at 16 km resolution, based on a square-root dependence of sky brightness on population.

Right (b): A map of the same area but with a linear dependence of the sky brightness on population. The qualitative differences between these models are considerable.

ical map of southern Ontario at 16 kilometer resolution, with each contour increment corresponding to a factor of two increase in brightness of the sky. The lowest contour on the screen was at a value of 32 S_{10} units, but in the figure the 128 unit level is the lowest discernible.

Figure 1b was produced by the same program, but using a brightness function which was *linearly* dependent on the population. The qualitative differences between this model and that depicted in figure 1a are extensive. Primarily, the larger cities dominate the linear model, while the "background" of small towns influences the square-root model (represented by equation (1)) the most. A linear model was originally used by Treanor (1973) in his work in Italy and by Walker (1973) for studies of California skies, but a brightness function which depends on the square root of population was found to more accurately fit the observational data in Ontario (Berry 1976). The actual nature of the population dependence may be a function of atmospheric as well as economic factors, so it is possible that the brightness function (1) will not be directly applicable in other geographic areas.

Figure 2 is a mosaic of photographs of the display, where the calculations have used a 4 km grid. It covers roughly the same area as figure 1, but at four times finer resolution. At this resolution, the model still is accurate near cities, and the individual towns can be resolved, which is not the case in figure 1a. At resolutions much less than 4 km, the model begins to break down significantly. Figure 3, at 2.5 km resolution, is centred on Toronto, and several inaccuracies can be detected. Between Toronto and Mississauga, for example, the display shows a brightness drop to about 2000 S_{10} units, arising because the cities are treated as point sources of light. Of course, the

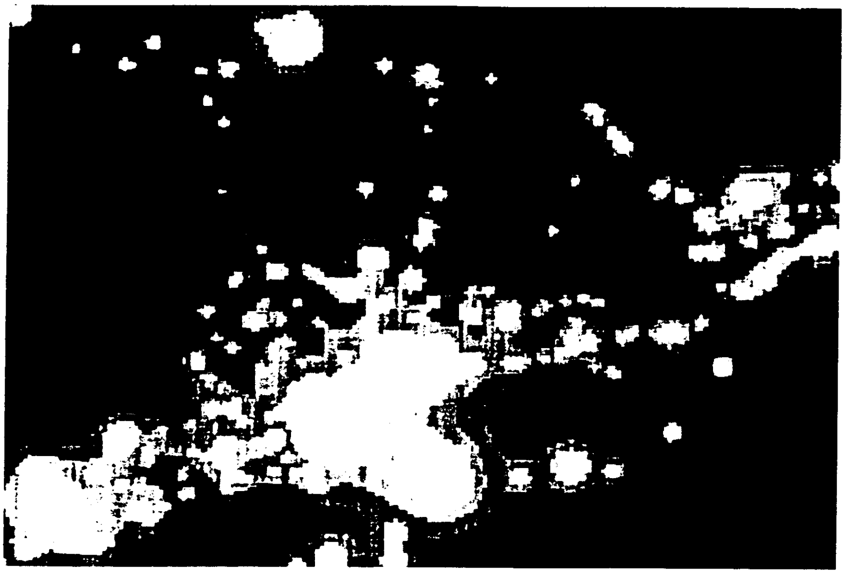

FIG. 2—Southern Ontario light pollution at 4 km resolution. Comparing this with figure 1b shows that the smaller towns are significant contributors to the background in the square-root model adopted in the work on Ontario.

cities are extended sources merging with each other, and the brightness actually stays around 8000 between the cities.

Although the brightness function (1) is based on simple assumptions, it represents fairly well the distribution of artificial sky light, and is thus a useful tool for attempting to understand and control the problem. It is easily adapted to study the future extent of light pollution in Ontario.

The Dynamic Model. If equation (1) is rewritten as

$$B = a\sqrt{p}\, Q(D), \tag{2}$$

it is clear that the brightness depends on three independent factors. The last, Q, is a function of the atmosphere and distance only, and may therefore be considered to be independent of time. To determine the year-to-year changes in the sky brightness, it is therefore sufficient to consider the changes in a and p.

It was decided to extend the static model back as far as the year 1950 and forward to the year 2000. Prior to 1950, the world wars and economic fluctuations make estimation of sky brightness at that time very difficult. Since 1950, the economic situation has changed fairly smoothly and mono-

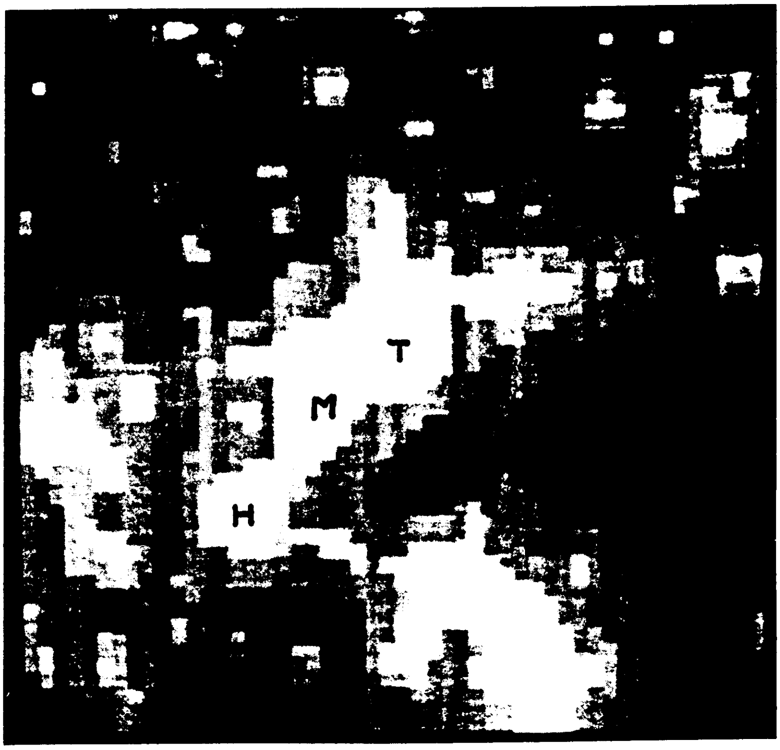

FIG. 3—The area around Toronto (T), Mississauga (M) and Hamilton (H), at 2.5 km resolution. At this level, the accuracy of the model begins to break down near cities. (The vertical streaks result from a "mistuned" display.)

tonically, so that it can be reasonably well represented by simple functions of time.

To determine the growth of population, Canadian census figures for each city in 1951, 1961, and 1971were fitted by a second-order polynomial, each city being considered separately. In a large proportion of the cases, the second order term was very small relative to the linear term, so a linear growth curve would probably have also been acceptable. Exceptions were a few, mostly new, rapidly growing cities such as Mississauga. For some cities, extrapolation of a second-order curve to the year 2000 would have given absurd results. In these cases, and in the U.S., where only two sets of census figures were available, a linear growth curve was used. In the U.S., however, the population has stabilized so well that no significant error would have resulted even from the assumption of a constant population for the American cities.

Many of the populations predicted by these simple curves may seem unrealistic. It should be noted, however, that the effect of errors in population is quite unimportant. If, for example, the populations of all towns were over-estimated by a factor of two the resulting brightness increase would be less than 0.4 magnitudes, or about the limit of accuracy of the model given by (1).

The changes in the value of a are difficult to determine accurately, as measurements of sky brightness have only been made in the last few years. Qualitative estimates by observers (Hogg 1975, Thompson 1976) suggest an increase of 7–10% per year. Riegel (1973) states that the rate of increase of outdoor public lighting was "an astonishing 23 per cent per year between 1967 and 1970". This figure and an exponential dependence of a on time, a reasonable model for short-term growth, leads to unrealistic results when extended more than a few years. A figure of 10% per year (including both the growth of a and $\sqrt{p}$) was chosen for the model, because it agrees well with the early qualitative observations, and because it leads to believable results. The increase is rapid enough that it is a reasonable approximation to the current trend in growth, but not so fast that it cannot be applied over a fairly large time span. It may still be conservative, however. Since the growth of $\sqrt{p}$ in Ontario is about 1.6% per year, and the total growth of 10% had been adopted, the growth of a was set at 8.4% per year, with

$$a(t) = a_0 \, 1.084^{(t-t_0)} \tag{3}$$

The origin, t_0, was set at 1975 with a_0 equal to 50.0 S_{10} units per root person.

To generate a map for a given year, the same procedure was used as for the static (1975) model, but with the time-dependent values of the a constant and populations instead of the fixed ones. Doing this for the years 1950–2000 in steps of one year gave a set of 51 maps. Figures 4 and 5 present 11 of these maps, at 5 year intervals. In figure 4, the maps for the years 1950–1975 are given, while those for 1975–2000 are in figure 5. The former can be compared with comments on sky quality by observers during the past years, and with the few scattered measurements made. The agreement between these is good.

Extrapolating to the year 2000 involves a number of assumptions which may not be reasonable. The most obvious is that it assumes that growth trends will continue in the same manner as for the last 25 years. Due to restrictions in energy and funds, it is likely that the growth of light pollution will level off in the near future. The author feels that this may begin about 1985. This prediction is based on parallelism with population trends in northern New York, where an influx of people from the rural areas to the urban, such as is occurring in Ontario now (see Table I), was followed by

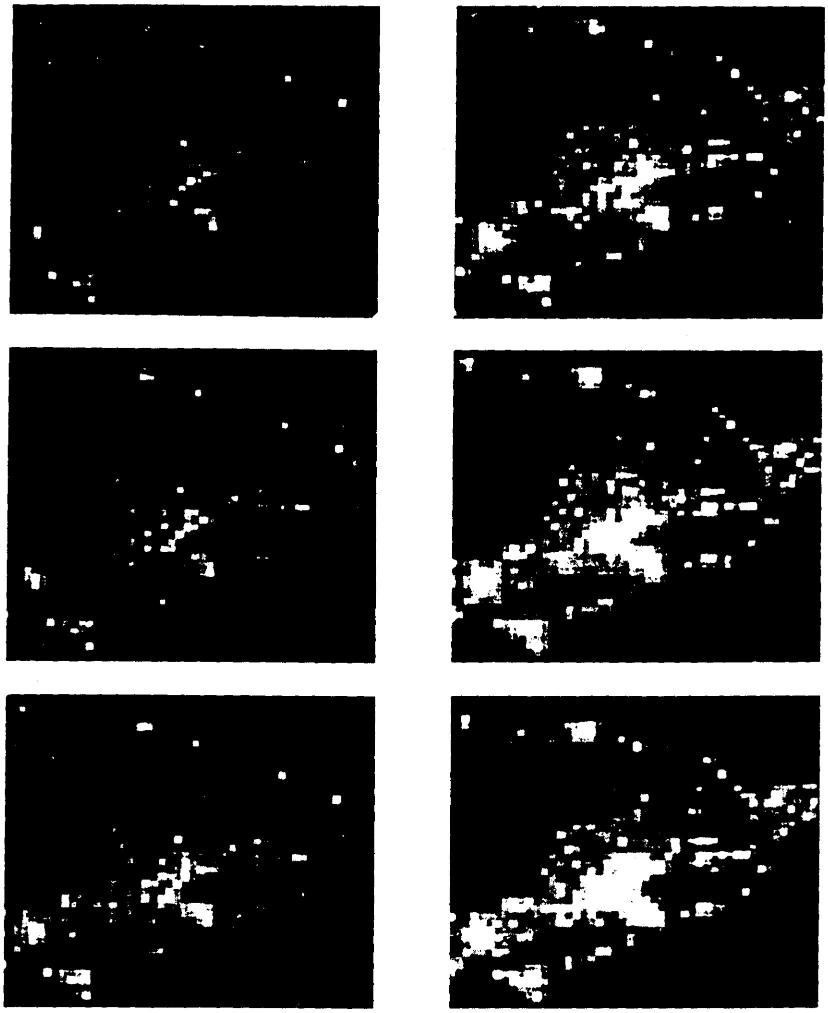

FIG. 4—The growth of light pollution, at 5-year intervals, in southern Ontario. From top left to bottom right, the maps are for 1950, 1955, 1960, 1965, 1970 and 1975.

a rapid decline in growth. The populations of cities such as Buffalo are now very stable, and it is possible that the same sort of phenomenon will occur in Toronto and the neighbouring communities. This decline in growth would be accompanied by a drop in housing construction, and then perhaps by a drop in the installation and "upgraded replacement" rates of street lamps. Because such a sequence of events could occur in the near future (i.e. before

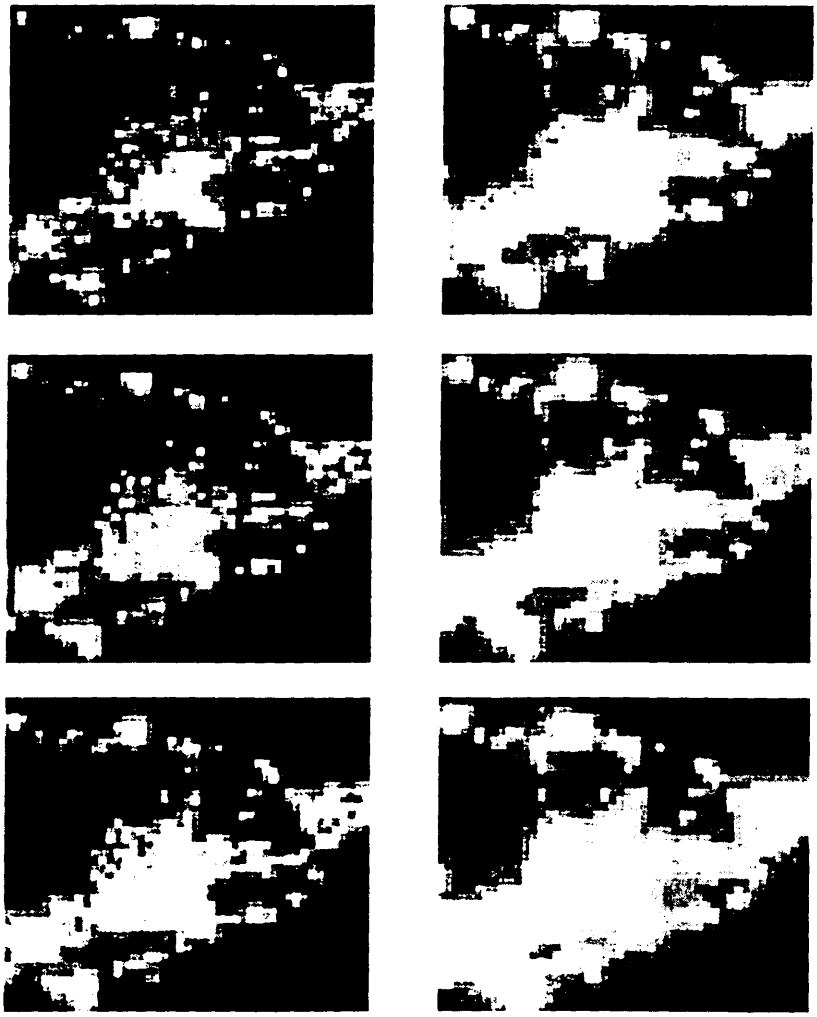

FIG. 5—The growth of light pollution, at 5 year intervals, in southern Ontario. From top left to bottom right, the maps are for 1975, 1980, 1985, 1990, 1995 and 2000. Note that once the sky brightness at a site reaches 250 S_{10} units, the lowest level in the figure, it is ruined as an observing site within a decade.

2000), the maps for the next 25 years should really be regarded as extrapolations rather than predictions.

In the year 2000, the model predicts that almost the entire inhabited land mass of Ontario will have a sky brightness of at least 2000 S_{10} units. At this brightness, the Milky Way is invisible. Aside from the obvious implications

TABLE I

GROWTH OF POPULATION IN ONTARIO

Year	Urban	Total	Urban/Total
1951	2,894,921	4,597,542	0.630
1961	4,194,482	6,236,092	0.673
1971	5,809,986	7,703,106	0.754
1981	7,741,434		
1991	9,988,825		
2001	12,552,160		

The growth of population in Ontario, from the census figures of 1951, 1961 and 1971 extrapolated to 2001. Note the increase in the number of people in the cities relative to those in rural locations.

for astronomy in the area, this would mean that the aesthetic quality of a country sky would be lost forever.

Thus, future observing prospects are poor, even with the conservative growth rate of 10% per year. There are a number of areas, though, particularly near rapidly growing towns, where the growth rate is much higher. The David Dunlap Observatory, located a few miles from Richmond Hill near Toronto, is experiencing a 27% per year growth rate in sky brightness (Bolton 1975)! Such areas, on the "advancing edge" of a highly populated zone, are suffering extreme, but probably short term, increases in the night sky brightness. Growth rates of 20% or more are not uncommon in the outlying suburbs of Toronto. Even though the results presented in figure 5 seem extreme, they are based on a conservative growth rate. The real long-term situation may be much worse.

Even as recently as 1973, serious underestimates of the future extent of the problem have been published. Walker (1973) uses some results from Riegel to determine that we may expect the light intensity "per capita" to eventually increase to 2.4 times its 1966 value (assuming no growth in the energy per capita devoted to lighting, only the conversion to more efficient light sources). This growth rate, of course, is that of our *a* parameter. At the (conservative) increase of 8.4% per annum, it would take 12 years to raise the level of light pollution by a factor of 2.4. This would imply that light pollution would be population-growth limited by 1978, but there is certainly no hint yet of the sky brightness increase levelling off. Based on the measured 23% per year increase in the U.S., we would expect the sky brightness to have "saturated" in 1970 or 1971, which it has not done. Walker's attempt to place a limit on the per capita growth was based on reasonable assumptions, but the message is clear – there is no apparent limit to how bright the night sky will get before its brightening is checked. It would be reassuring indeed to find that the sky brightness "per root capita" would never increase more than a factor of 10 over its present value. This roughly corresponds to the map for the year 2000 in figure 5. It should be noted that the area included in that map is about an order of magnitude less populated than similar areas in New England, so the situation in parts of the U.S. is likely to be even worse.

Conclusions. Using a simple mathematical model for the scattering of light in the atmosphere, augmented by a fairly small number of observations, it is possible to simulate the light pollution situation in Ontario. Graphics-oriented computers are well suited to providing easily interpreted and effective demonstrations of the problem. Programs written to simulate the static situation can be easily adapted to extrapolate past trends in population growth and public lighting in order to predict what the skies will be like in the next few years.

Maps showing the extrapolated sky brightness distributions for the next 25 years illustrate dramatically that the observing prospects in developed areas of the country will become very poor. As they are based, however, on assumptions which are probably conservative, the true future situation will likely be much worse. The initial assumptions used to produce these maps were based on very scanty observations of light pollution in the past, and on a few highly subjective qualitative measurements by observers in the area. If observatories had been monitoring the sky brightness on a regular basis, as the Toronto Centre's Sky Brightness Programme has begun to do, these assumptions could have been more accurate and the extrapolations would therefore have been even more reliable and useful. It is hoped that a study such as this may be made in a decade or so with sufficient information available to provide precise information on the location of the best skies in the following years.

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